

The Tangent Ratio

Lesson Objective INBAT use the tangent ratio to find unknown side lengths

Trigonometric Ratio: A trigonometric ratio is a ratio of the lengths of two sides in a right triangle.

All right triangles with a given acute angle are similar by AA ~.

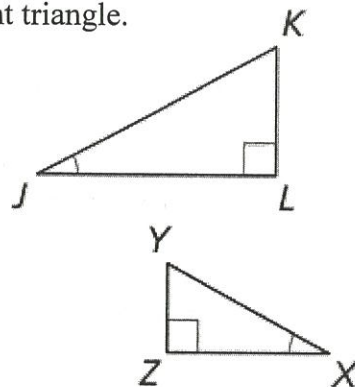
In the diagram at right, $\triangle JKL \sim \triangle XYZ$. So, you can write the following ratio of sides:

$$\frac{YZ}{KL} = \frac{ZX}{JL}$$

which can be rewritten as...

$$\frac{YZ}{ZX} = \frac{KL}{JL}$$

this is a trig ratio!
ratios compare 2 sides of $\triangle$



So, trigonometric ratios are constant for a given angle measure.

Tangent Ratio: The tangent ratio is a trigonometric ratio for acute angles that involves the lengths of the legs of a right triangle.

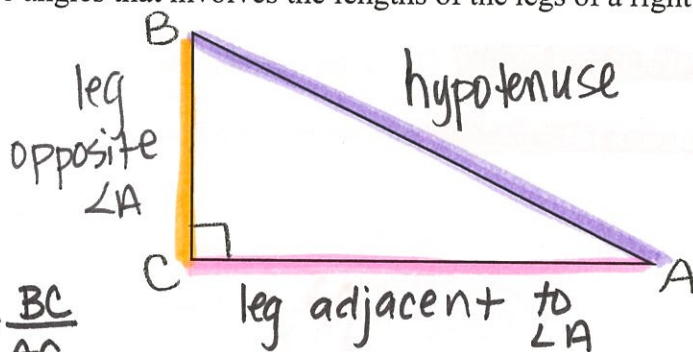
Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as $\tan A$) is defined as follows...

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

$$\rightarrow \tan A = \frac{BC}{AC}$$

$$\tan B = \frac{AC}{BC}$$

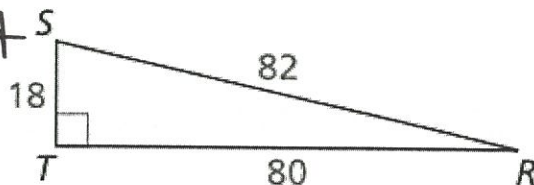


Example:

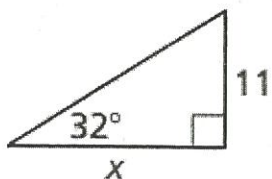
1. Find $\tan S$ and $\tan R$. Write each answer in EXACT form, and as a decimal rounded to four places.

$$\tan S = \frac{18}{18} = \frac{40}{9} \approx 4.4444$$

$$\tan R = \frac{18}{80} = \frac{9}{40} = 0.2250$$



2. Find the value of x . Round your answer to two decimal places.



$$\tan 32^\circ = \frac{11}{x}$$

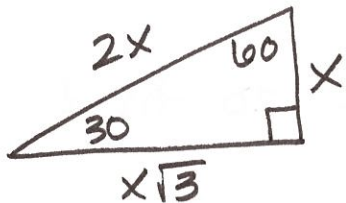
$$0.6249 = \frac{11}{x}$$

$$x(0.6249) = 11$$

$$x = \frac{11}{0.6249}$$

$$x \approx 17.6037$$

3. Use a special right triangle to find $\tan(60^\circ)$.

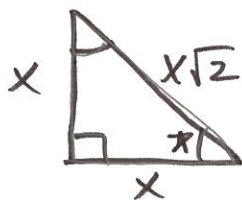


$$\tan(60^\circ) = \frac{x\sqrt{3}}{x} = \boxed{\sqrt{3}}$$

4. Use a special right triangle to find $\tan(30^\circ)$.

$$\tan(30^\circ) = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

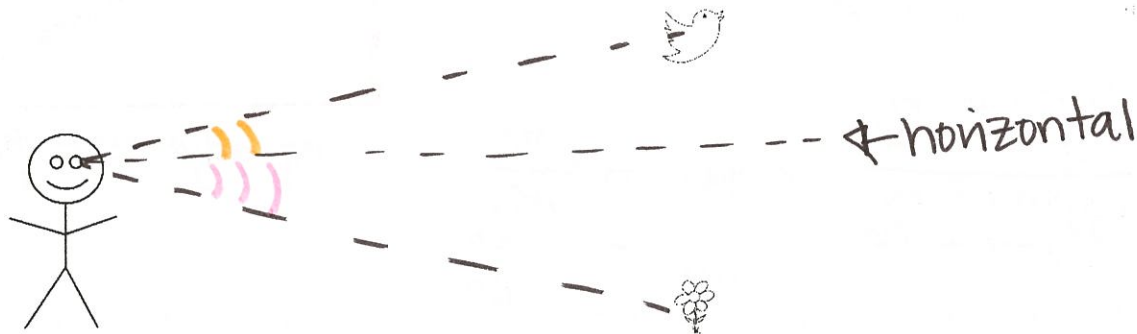
5. Use a special right triangle to find $\tan(45^\circ)$.



$$\tan(45^\circ) = \frac{x}{x} = \boxed{1}$$

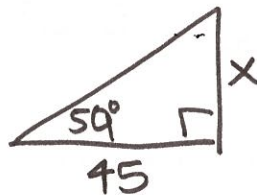
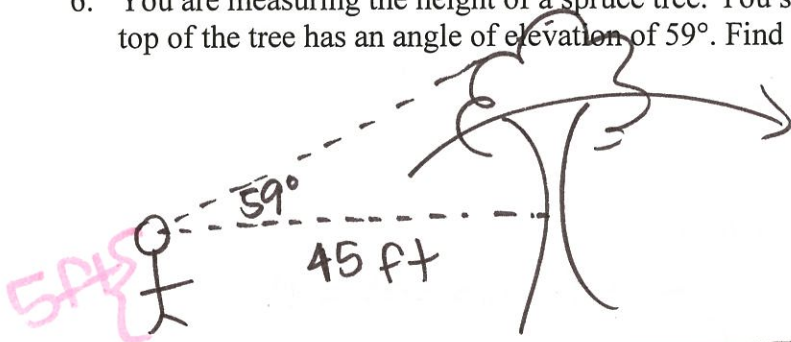
Angle of Elevation: The angle that an upward line of sight makes with a horizontal line

Angle of Depression: The angle that a downward line of sight makes with a horizontal line



Example:

6. You are measuring the height of a spruce tree. You stand 45 feet from the base of the tree. Your view to the top of the tree has an angle of elevation of 59° . Find the height of the tree to the nearest foot.



$$\tan 59^\circ \approx 1.6643$$

$$\frac{1.6643}{1} = \frac{x}{45}$$

$$x = 45(1.6643)$$

$$x \approx 74.8926$$

The tree is ≈ 79.8926 ft
 ≈ 80 ft